Exam 3

Math 108, Section 1
Due: Monday, December 7, 2009 at 11am

This exam is an individual assignment. You are permitted to ask questions of your instructor only; collaboration of any other kind is not allowed and will be considered an act of academic dishonesty and prosecuted to the fullest extent. Your signature in the blank next to "name" below indicates that you agree to abide by the principles of academic honesty on this exam. Submit this signed cover page with your exam. There are 100 points possible.

Name: ________________________________

1. (10 points) Suppose R and S are relations on A={1,2,3}. Construct relations R and S so that $R \circ S \neq S \circ R$. (Thus, proving relation composition is not commutative.)
2. (a) (5 points) Write a useful denial of the statement $(x, y) \in A \times B$.
   (b) (10 points) Prove $(A \times B) - (A \times C) = A \times (B - C)$
3. (10 points) Prove $(A - B) \times (C - D) \subseteq (A \times C) - (B \times D)$
4. (10 points) Let A and B be nonempty sets.
   (a) Prove $A \times B = B \times A$ if and only if $A = B$.
   (b) Is the statement in part (a) true if one of A or B is empty? Give reasons for your answer.
5. (10 points) Let R be a relation on the set A. Prove R is transitive if and only if $R \circ R \subseteq R$.
6. (15 points) A relation R on $\mathbb{Z}$ is given by xRy if and only if 3 divides $x - y$.
   (a) Show that R is an equivalence relation on $\mathbb{Z}$
   (b) Find the equivalence classes 0/R, 1/R, 2/R and 3/R
   (c) Compare 0/R to 3/R. What do you notice?
7. (15 points) A relation R on the set A is called circular if for all $x, y \in A$, xRy and yRz implies zRx. Prove that if R is reflexive and circular, then R is an equivalence relation.
8. (15 points) A relation R on $\mathbb{R} \times \mathbb{R}$ is given by $(a, b)R(c, d)$ if and only if $a + d = b + c$
   (a) Show that R is an equivalence relation on $\mathbb{R} \times \mathbb{R}$
   (b) Find the equivalence class (7,3)/R
   (c) Give the partition on $\mathbb{R} \times \mathbb{R}$ induced by R.